Roll No: $\square$
BTECH
(SEM II) THEORY EXAMINATION 2021-22

## ENGINEERING MATHEMATICS-II

Time:3 Hours
Total Marks:100

## Notes-

- Attempt all sections and assume any missing data.
- Appropriate marks are allotted to each question, answer accordingly.

| SECTION -A |  | Attempt all of following question in brief | Marks $(\mathbf{1 0} \times \mathbf{2}=\mathbf{2 0})$ |
| :--- | :--- | :--- | :---: |
| Q.1(a) | Cind the inverse of the matrix $A=\left[\begin{array}{ll}4 & 3 \\ 5 & 7\end{array}\right]$. | 1 |  |
| Q.1(b) | For what value of ' $b$ ' the rank of the matrix $A=\left[\begin{array}{ccc}1 & 5 & 4 \\ 0 & 3 & 2 \\ b & 13 & 10\end{array}\right]$ is 2. | 1 |  |
| Q.1(c) | Show that $f(z)=z+2 \bar{z}$ is not analytic anywhere in the complex plane. | 2 |  |
| Q.1(d) | Define a harmonic function and conjugate harmonic function. | 2 |  |
| Q.1(e) | Find the unit normal vector to the surface $z=x^{2}+y^{2}$ at the point $(1,1,2)$. | 3 |  |
| Q.1(f) | Find the value of 'a'for which the vector field $\vec{v}=a(x+y) \hat{\imath}+4 y \hat{\jmath}+3 \hat{k}$, is solenoidal. | 3 |  |
| Q.1(g) | Find the constant term when $f(x)=1+\|x\|$ is expanded in Fourier series in the interval $(-3,3)$. | 4 |  |
| Q.1(h) | State Dirichlet's condition for the expansion of $\mathrm{f}(\mathrm{x})$ in Fourier series. | 4 |  |
| Q.1(i) | Classify: $\frac{\partial u}{\partial t}=c^{2} \frac{\partial^{2} u}{\partial x^{2}}$. | 5 |  |
| Q.1(j) | Find the general solution of $\frac{\partial u}{\partial x}=\frac{\partial u}{\partial y}$ using method of separation of variables. | 5 |  |


|  | ION -B |  | Marks (3×10=30) | CO |
| :---: | :---: | :---: | :---: | :---: |
| a) | Find the value of k , such that the system of equations $4 x+9 y+z=0, k x+3 y+k z=0$ and $x+4 y+2 z=0$ Has non-trivial solution. Hence find the solution of the system. |  |  |  |
| Q.2(b) | If $f(z)=u+i v$ is an analytic function, find $f(z)$ in term of $z$ if $\boldsymbol{u}-\boldsymbol{v}=\frac{e^{y}-\cos x+\sin x}{\cosh y-\cos x}$ When $f\left(\frac{\pi}{2}\right)=\frac{3-i}{2}$. |  |  | 2 |
| Q | Use Divergence Theorem to evaluate $\iint_{\widehat{S}} \vec{F} d \vec{S}$ where $\vec{F}=4 x \hat{\imath}-2 y^{2} \hat{\jmath}+z^{2} \hat{k}$ and S is the surface bounding the region $x^{2}+y^{2}=4, z=0$ and $z=3$. |  |  |  |
| Q. | For the Fourier series for the function given by $f(x)=\left\{\begin{array}{cc}2 t & , 0<t<1 \\ 2(2-t), & 1<t<2\end{array}\right.$ |  |  |  |
| Q.2(e) | A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y=y_{0} \sin ^{3} \frac{\pi x}{l}$. If it is released from rest from this position, find the displacement $y(x, t)$. |  |  | 5 |

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| SECTION -C |  | Attempt any one of the following questions | Marks $(\mathbf{1} \times \mathbf{1 0}=\mathbf{1 0})$ | CO |
| :--- | :--- | :---: | :---: | :---: |
| Q.3(a) | Verify Cayley-Hamilton Theorem for the matrix $A=\left[\begin{array}{lll}1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1\end{array}\right]$. Hence evaluate $A^{-1}$. | 1 |  |  |
| Q.3(b) | Find the Eigen values and Eigen vectors of the matrix $A=\left[\begin{array}{cc}2 & -2 \\ 1 & 1\end{array}\right]$ |  |  |  |
| 1 | 3 | -1 |  |  |$] . .$|  |
| :---: |


| SECTION -C |  | Attempt any one of the following questions | Marks (1×10=10) | CO |
| :---: | :---: | :---: | :---: | :---: |
| Q.4( | If $f(z)$ is a regular function of $z$, prove that $\left(\frac{\partial^{2}}{\partial x^{2}}+\frac{\partial^{2}}{\partial y^{2}}\right)\|f(z)\|^{2}=4\left\|f^{\prime}(z)\right\|^{2}$. |  |  | 2 |
| Q.4(b) | An electrostatic field in the $x y$ - plane is given by the potential function $\varphi=3 x^{2} y-y^{3}$, find the stream function and hence find complex potential. |  |  | 2 |


| SECTION -C |  | Attempt any one of the following questions | Marks $(\mathbf{1} \times \mathbf{1 0}=\mathbf{1 0})$ | CO |
| :--- | :--- | :--- | :---: | :---: |
| Q.5(a) | State and verify Green's theorem in the plane for $\phi\left(3 x^{2}-8 y^{2}\right) d x+(4 y-6 x y) d y$ where C is <br> the boundary of the region bounded by $x \geq 0, y \leq 0$ and $2 x-3 y=6$. | 3 |  |  |
| Q.5(b) | If the directional derivative of $\varphi=a x^{2} y+b y^{2} z+c z^{2} x$ at the point $(1,1,1)$ has maximum <br> magnitude 15 in the direction paratlel to the line $\frac{x-1}{2}=\frac{y-3}{-2}=\frac{z}{1}$, find the values of $\mathrm{a}, \mathrm{b}$ and c. | 3 |  |  |


| SECTION -C |  | Attempt any one of the following questions | Marks $(\mathbf{1} \times \mathbf{1 0}=\mathbf{1 0})$ |
| :--- | :--- | :--- | :---: |
| Q.6(a) | CO |  |  |
|  | Obtain Fourier series for the function $f(x)=\left\{\begin{array}{c}1+\frac{2 x}{\pi}, \quad \pi<x<0 \\ 1-\frac{2 x}{\pi}, 0<x<\pi\end{array}\right.$. | 4 |  |
|  | Hence deduce that $\frac{1}{1^{2}}+\frac{1}{3^{2}}+\frac{1}{5^{2}} \cdots \cdots \cdots=\frac{\pi^{2}}{8}$. |  |  |


| SECTION -C |  | Attempt any one of the following questions $\quad$ Marks $(\mathbf{1} \times \mathbf{1 0 = 1 0 )}$ | CO |
| :--- | :--- | :---: | :---: |
| Q.7(a) | Use the method of separation of variables to solve the equation $\frac{\partial^{2} u}{\partial x \partial t}=e^{-t} \cos x$ given that $u=$ <br> 0 when $t=0$ and $\frac{\partial u}{\partial t}=0$ when $x=0$. | 5 |  |
| Q.7(b) | Find the temperature distribution in a rod of length 2 m whose end points are fixed at <br> temperature zero and initial temperature distribution is $f(x)=100 x$. | 5 |  |

